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STUDY OF THE PROBLEMS OF TAKEOFF OF A SPACECRAFT FROM THE SURFACE OF MARS

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16. Abstract

A two-stage spacecraft is considered with a liquid-propelled rocket which takes off from the Mars surface into a predetermined Keplerian orbit. A dimensionless system of motion equations is offered, permitting, with the use of generalized dimensionless similitude coefficients, analysis of the spacecraft takeoff from planets. Problems of selecting the spacecraft takeoff trajectories for various elliptic and hyperbolic orbits are considered, and ballistic parameters of the spacecraft are determined. The optimum program of the two-stage spacecraft orbit insertion is shaped approximately. The calculated parameters are analyzed from the viewpoint of the maximum relative useful load for the piecewise-linear pitching program.

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STUDY OF PROBLEMS OF TAKEOFF OF A SPACECRAFT FROM THE SURFACE OF MARS

V.N. Borovenko and Z.I. Rumynskaya

In works [1, 2] problems of the takeoff of a spacecraft from the surface of Mars are discussed in connection with the development of flights to this planet. A characteristic of these works is the discussion of one-stage injection into low circular or elliptical orbits. In this work a more general approach is given to the analysis of conditions of takeoff of a spacecraft from the surface of Mars. /836*

A dimensionless system of equations of motion is proposed, which, by means of generalized dimensionless coefficients of similitude, allows an analysis of the takeoff of spaceships from planets to be made.

Problems of the choice of trajectories of takeoff of a spaceship into various elliptical or hyperbolic orbits are discussed, and the ballistic parameters of a flight vehicle are determined. An optimum program for injection of a two-stage spacecraft is approximately formulated. An analysis of the design-ballistic parameters of the spacecraft is made from the point of view of the maximum relative to the payload for the piecewise linear program of pitching.

The surface nature of the critical function in a multidimensional problem on a plane of the control parameters for a linear pitching program is shown.

1. Posing the Problem

We shall discuss a two-stage spacecraft with a liquid-fuel

*Numbers in the margin indicate pagination in the foreign text.

rocket engine, making a takeoff from the surface of Mars into the prescribed Kepler orbit. It is proposed that the field of gravitation of the non-revolving planet is central Newtonian. The atmosphere of the planet was accepted as homogeneous isothermic, with the average amounts of its parameters [3] as follows: $\rho_0 = 1.3 \cdot 10^{-5}$ g/cm³, $\beta = 0.09$ km^{-1}, where ρ_0 is the density of the atmosphere near the surface of Mars; β is the sign of the exponent in the law of change in density with altitude.

The thrust of the engines is limited; the mass of the spacecraft in the active sector of the trajectory changes according to an arbitrary law. The aerodynamic coefficients C_{x_0} and C_y^{α} are considered given.

The equations of motion of the spacecraft may be written in the form [4]:

where **v** is the velocity vector of motion; **r** is the radius vector of the spacecraft in the adopted system of coordinates; m and m_{sec} are the mass of the spacecraft and its flow rate; **R** and **P** are the resultant of all external forces and the thrust force of the engines respectively.

We transform the equations of motion (1) to dimensionless variables $\bar{v} = v/c$, $\bar{r} = r/R_0$, $\tau = G/G_0$, where c is the velocity of gas effluxes from the nozzle of the engines, R_0 is the radius of the planet, τ is the dimensionless flight time representing the ratio of the current weight of the burnt-out fuel G_f to the starting weight of the stage of the spacecraft, G_0.

Projecting equations (1) onto the axes of the starting system of coordinates and transferring from a derivative with respect to time to a derivative with respect to τ, we obtain: /837

$$\frac{d\bar{v}}{d\tau} = \frac{\cos\alpha}{1-\tau} - \frac{1}{pn_0}\left[\frac{1+\bar{y}}{\bar{r}^3}\sin\theta + \frac{\bar{x}}{\bar{r}^3}\cos\theta + A_{1x}\frac{\bar{v}^2 e^{-A_2\bar{h}}}{1-\tau}\right],$$
$$\frac{d\theta}{d\tau} = \frac{1}{\bar{v}}\left\{\frac{\sin\alpha}{1-\tau} - \frac{1}{pn_0}\left[\frac{1+\bar{y}}{\bar{r}^3}\cos\theta - \frac{\bar{x}}{\bar{r}^3}\sin\theta - A_{1y}\frac{\alpha\bar{v}^2 e^{-A_2\bar{h}}}{1-\tau}\right]\right\}, \qquad (2)$$
$$\frac{d\bar{x}}{d\tau} = \frac{A_3}{pn_0}\bar{v}\cos\theta, \quad \frac{d\bar{y}}{d\tau} = \frac{A_3}{pn_0}\bar{v}\sin\theta,$$
$$\bar{r} = \sqrt{\bar{x}^2 + (1+\bar{y})^2}, \quad \bar{h} = \bar{r} - 1,$$

where $\bar{x}$, $\bar{y}$ is the dimensionless distance and altitude of flight, reckoned in the starting system of coordinates; $\bar{h}$ is the dimensionless local flight altitude; θ is the angle between the velocity vector and the axis OX of the starting system of coordinates; α is the attack angle of the spacecraft; p is the control of the engine thrust.

The system of equations of motion (2) obtained is a function of five coefficients of similitude.

$$A_{1x} = \frac{c_{x_0}S}{2G_0}\rho_0 c^2 + kA_{1y}\alpha^2,$$
$$A_{1y} = \frac{c_y^{\alpha}S}{2G_0}\rho_0 c^2, \quad A_2 = \beta R_0, \quad A_3 = \frac{c^2}{g_0 R_0}, \qquad (3)$$
$$n_0 = \frac{P}{G_0}.$$

The scheme for injecting the spacecraft into elliptical orbits is shown in Fig. 1.

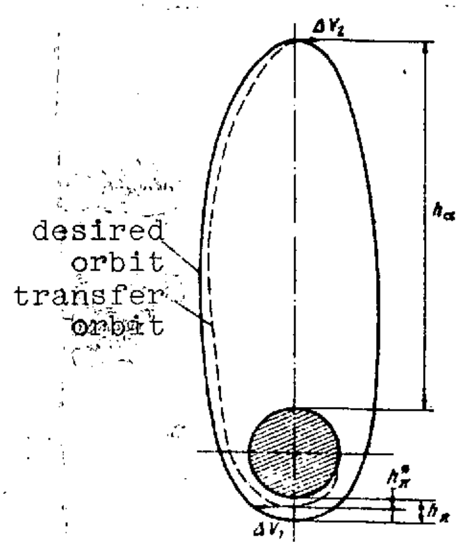

Fig. 1

The altitude of the apocenter $\bar{h}_\alpha$ of the transitional ellipse coincides with the amount necessary for final orbit, while the altitude of the pericenter h^*_π is chosen in the process of optimization. The desired elliptical orbit into which the injection of the spacecraft is considered is determined by the altitude of the apocenter $\bar{h}_\alpha$ and the altitude of the pericenter h_π.

The transition of the spacecraft into hyperbolic trajectories of takeoff is studied indirectly from the trajectory of injection. For this the hyperbolic orbit is prescribed only by the value of the velocity at infinity v_∞.

In accordance with the adopted scheme for injection of the parameter of the trajectory at the end of the sector of injection, we choose from the condition of the maximum of the payload $\bar{G}$ which is being put into orbit.

The velocity at the end of the section of injection must satisfy the condition:

$$\bar{v}_k = \sqrt{\frac{2\bar{\mu}}{\bar{r}_k} + \bar{H}} + \Delta\bar{v}_1, \tag{4}$$

where $\bar{\mu}$ is the gravitational constant of Mars; $\bar{r}_k$ is the planetocentric distance at the moment of completion of the active leg; $\bar{H}$ is the constant integral of energy of Kepler motion.

$$\bar{H} = \begin{cases} -\dfrac{2\bar{\mu}}{\bar{r}_a + \bar{r}_\pi^*}, \quad \bar{r}_\pi^* = 1 + \bar{h}^*(\bar{r}_k, \theta_k, \bar{v}_k) & \text{for elliptical orbits} \quad (5) \\ \bar{v}_\infty^2 & \text{for hyperbolic orbits} \quad (6) \end{cases}$$

$$\Delta\bar{v}_1 = \frac{A_{1k}}{A_3(1-\tau_{k1})} \int_{\bar{h}_k}^{\bar{h}_a} v \frac{e^{-A_2\bar{h}}}{\sin(\vartheta - \theta)} d\bar{h}$$

is the increase in velocity necessary /838
for overcoming resistance in passive flight in the limits of the atmosphere; $\bar{h}_a$ is the altitude of the boundary of the atmosphere of Mars; ϑ is the angle between the initial and current radius vectors of the spacecraft, τ_{ks} is the dimensionless period of work of the engine of the first stage.

In injection into elliptical orbits in the apocenter of the transitional ellipse, an impulse $\Delta\bar{v}_2$ is given, which is necessary for increasing the altitude of the pericenter of the ellipse from $\bar{h}_\pi^*$ the required amount of $\bar{h}_\pi$

$$\Delta\bar{v}_2 = \sqrt{\frac{2\bar{\mu}\,\bar{r}_\pi}{\bar{r}_a(\bar{r}_\pi + \bar{r}_a)}} - \sqrt{\frac{2\bar{\mu}\,\bar{r}_\pi^*}{\bar{r}_a(\bar{r}_\pi^* + \bar{r}_a)}}. \tag{7}$$

The impulses $\Delta\bar{v}_1$ and $\Delta\bar{v}_2$ increase τ_{k2}, which determines the relative payload put into orbit.

In injection into hyperbolic orbits $\Delta \bar{v}_2 = 0$, at the end of the active leg beyond the bounds of the atmosphere $\Delta \bar{v}_1 = 0$.

Thus, the following problem is posed: in the class of piecewise-continuous control functions, $-\pi/2 \leqslant \varphi(\tau) \leqslant \pi/2,\ 0 \leqslant p(\tau) \leqslant 1$ and of ballistic parameters of the spacecraft $\tau_{k1},\ n_{01},\ n_{02},$ which satisfy the boundary conditions of the problem, to find functions and parameters such that

$$\bar{G} = \bar{G}(\tau, \bar{v}, \theta, \bar{x}, \bar{y}, \varphi(\tau), p(\tau), \tau_{k1}, n_{01}, n_{02}, \tau_{k2})\Big|_{\bar{v}=\bar{v}_k} = \max_{\{\varphi(\tau),\, p(\tau),\, \tau_{k_1},\, n_{01},\, n_{02}\}} \bar{G}. \tag{8}$$

This problem belongs to the class of variational problems. We will solve it by direct methods of optimization.

2. Selection of the Ballistic Parameters of a Spacecraft

As is shown in [2], the optimal program of pitching in injection into orbit of a satellite of Mars is nearly piecewise-linear. We shall study the solution of problem (8) with the following program of pitching:

$$\varphi(\tau) = \begin{cases} \dfrac{\pi}{2}, & \text{if } \tau \leqslant \tau_0, \\ \dfrac{\pi}{2} - \left(\dfrac{\pi}{2} - \varphi_{k1}\right)\dfrac{\tau - \tau_0}{\tau_{k1} - \tau_0}, & \text{if } \tau_0 < \tau \leqslant \tau_{k1}, \\ \varphi_{k1} + \varphi_2'\tau, & \text{if } \tau = \tau_2, \end{cases} \tag{9}$$

$$\alpha(\tau) = \varphi(\tau) - \theta(\tau). \tag{10}$$

Furthermore, we assume $p(\tau) = 1$.

In formula (9) we designated: τ_0 is the length of the vertical sector of the trajectory; τ_{k1} is the duration of work of the first stage; φ_{k1} is the final angle of pitching of the first stage of the spacecraft; φ_2 is the angular rotation velocity of the longitudinal axis of the second stage. We shall determine the amount of relative payload by the expression:

$$\bar{G} = \prod_{i=1}^{2} (1 - (1 + a_i)\tau_{ki} - \gamma_i n_{0i}), \tag{11}$$

where a_i are the weight coefficients of the structure; γ_i are the specific weights of the engines of the spacecraft.

The amounts of these coefficients are determined by the parameters of the spacecraft n_{0i}, τ_{ki}, and they also depend on the type of fuel [5].

Taking into consideration the free constants of the control law and the parameters of the spacecraft, the relative payload put into Keplerian orbit is a function of five parameters:

$$\bar{G}(\tau_{k1}, n_{01}, \varphi_{k1}, n_{02}, \varphi_2'). \tag{12}$$

We will survey the influence of these parameters on the amount of relative payload when the spacecraft is put into elliptical orbit with h_π = 1,000 km, h_α = 14,800 km.

In Fig. 2 sections of the surface G $(\tau_{k1}, n_{01}, \varphi_{k1}, n_{02}, \varphi_2')$ with lines of equal level are presented. With this, on the fields of isolines the parameters of the program of pitching φ_{k1} and φ_2' were placed, while the ballistic parameters of the spacecraft for every point of the surface $\varphi_{k1}\varphi_2'$ were taken as optimal in the sense of (8).

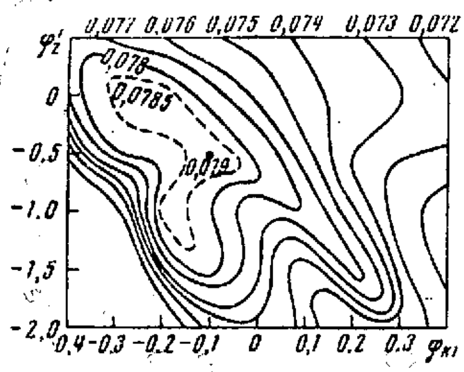

Fig. 2

The amounts of the coefficients of similitude (3), introduced into the calculation, correspond to a spacecraft, the data on which were presented in the article [1]. Fig. 2 shows that the dependence of the amount of relative payload $\bar{\bar{G}}$ on the parameters of the spacecraft and the program of pitching is very complex. This characteristic of the problem does not allow reliable results to be obtained by simple gradient methods of search for the extremum of a function of several variables.

More effective in solving the problem of optimization of parameters of a spacecraft according to the maximum of relative payload was the method of "gullies" [6].

In Fig. 3 the parameters of the trajectory of takeoff with an optimal distribution of parameters of the spacecraft are shown. The flight altitude in the process of injection has a maximum at the beginning of the work of the second stage, and then it decreases. It is known that this effect takes place also in injection of a spacecraft from the surface of the earth, but more weakly.

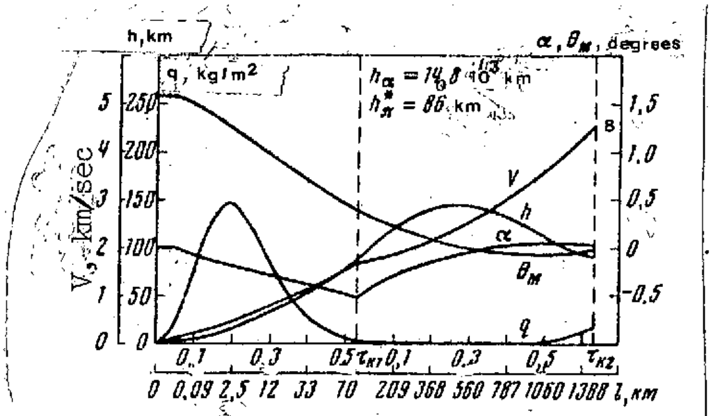

Fig. 3

Results of the calculations of the optimal parameters of the spacecraft in takeoff into elliptical orbits with h_π = 1,000 km and h_α = 1,000–30,000 km are presented in Fig. 4.

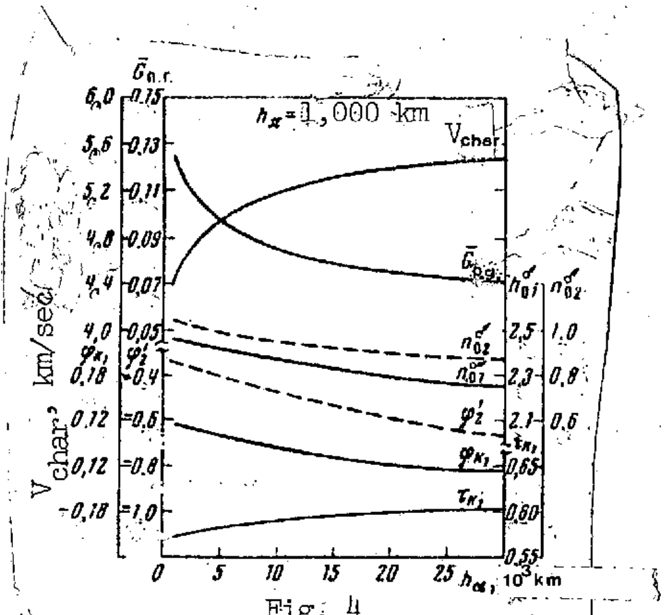

Fig. 4

An analysis of the surface G $(\tau_{k1}, n_{01}, \varphi_{k1}, n_{02}, \varphi_2')$ in injection into hyperbolic trajectories of the prescribed energy is of interest. A characteristic of these trajectories is the absence of an impulse $\Delta\bar{v}_2$.

Sections of the surface $\overline{G}$ $(\tau_{k1}, n_{01}, \overline{\varphi}_{k1}, n_{02}, \varphi_2')$ with isolines plotted onto the surface of the parameters of the program of pitching φ_{k1}, φ_2' with optimal amounts of parameters of the spacecraft $\tau_{k1}, n_{01}, n_{02}$ are shown in Fig. 5.

Hyperbola V_∞ = 20km/sec

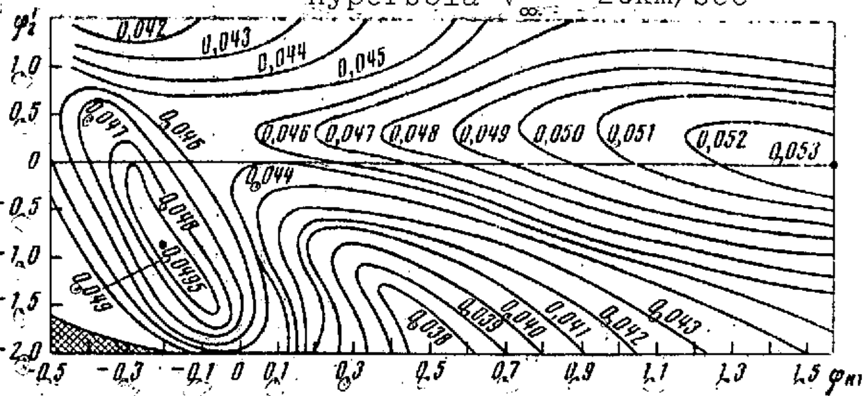

Fig. 5

The results obtained indicate the presence of two classes of optimal trajectories of injection into hyperbolic trajectories. The optimum located in the region $\varphi_{k1} = -0{,}2$, $\varphi_2' = -0{,}9$, corresponds to inclined trajectories of injection, while the optimum, located in the region $\varphi_{k1} = \pi/2$, $\varphi_2' = 0$, corresponds to the vertical trajectories of injection. Vertical injection is more desirable than inclined trajection. The optimal parameters of the spacecraft, /841 corresponding to two classes of optimal trajectories of injection into hyperbolic trajectory with V_∞ = 2 km/sec, are presented in the table

Class of optimum	φ_{k1}	φ_2'	τ_{k1}	n_{01}	n_{02}	$\overline{G}$	V_{char} km/sec
1	—0,2	—0,9	0,6	2,4	0,5	0,0495	6,26
2	$\pi/2$	0	0,6	6,0	4,0	0,0530	5,82

In Fig. 6 the dependencies of the optimal parameters of the spacecraft on the amounts of V_∞ = 0-3 km/sec, corresponding to various hyperbolic trajectories of takeoff in class 1, are given.

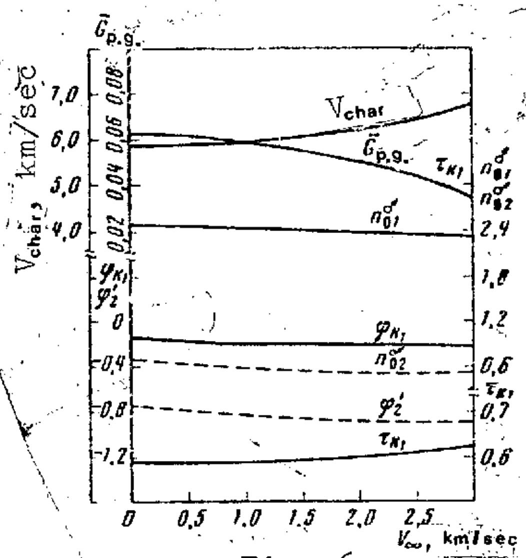

Fig. 6

REMARKS

1. As is seen from Figs. 2 and 5, the values of $\bar{G}$ in the region of the optimum change little in a variation in the parameters of the program of pitching in a certain range. An even weaker change of the relative payload takes place with a change in the thrust-to-weight ratio of the stages. This permits the parameters of the spacecraft to vary within a certain range of values, without changing the values of criterion of quality $\bar{G}$.

2. In selection of the optimal parameters of the spacecraft in the problem, limitations on the size of the angle of attack were not introduced. An analysis of the results obtained shows that the angles of attack at flight speeds close to that of sound in optimal trajectories are within the limits of from -5 to -7°. In addition, the velocity head does not exceed 80 kg/m².

In separation of the stages, the angles of attack go up to $-30°$, but this takes place practically beyond the bounds of the atmosphere. Thus, the optimal trajectories of injection may be realized.

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